Informational Role of Investment and Bankruptcy Costs

Ben Charoenwong, Hyunsoo Doh, Yiyao Wang*

November 21, 2018

Abstract

We study an informational role of investment when a bankruptcy market suffers from information asymmetry. In our model, equityholders of a levered firm forgo positive-NPV projects when the firm's asset quality is low. But due to this underinvestment, ill-informed potential asset buyers can distinguish between good firms and bad firms from their past investment decisions. Put differently, the agency friction between equity and debt can alleviate the information friction in the bankruptcy market. Therefore, policies seeking to stimulate investment during recessions may reduce the informational contents of investment, leading to the lower recovery value for assets in default. We provide some suggestive empirical evidence that supports our model.

Keywords: Debt overhang, information asymmetry, informative underinvestment, bankruptcy costs.

*Ben Charoenwong is at National University of Singapore; Email: ben.charoenwong@nus.edu.sg. Hyunsoo Doh is at Nanyang Technological University; Email: hdoh@ntu.edu.sg. Yiyao Wang is at University of Chicago; Email: yiyaowang@chicagobooth.edu. This paper has been previously circulated under the title "Debt Overhang and Asymmetric Information in the Secondary Market." We thank Hengjie Ai, Heitor Almeida (discussant), Markus Brunnermeier, Simba Chang, Hui Chen, Zhiguo He, Chuan-Yang Hwang, David Reeb, Hongda Zong (discussant) for their helpful comments. Any remaining errors are ours.

1 Introduction

Following recessions, various fiscal policy measures have been used by the governments to stimulate the economy. One of such policy tools is to provide corporate tax-based incentives to new investments. For example, during the 2001 and 2008 recessions, the US government granted more investment tax credits to foster investments, which allowed firms to deduct a larger percentage of their investment related costs from their taxable income.

Existing studies argue that the underinvestment problem caused by debt overhang can justify such government interventions. Specifically, as addressed by Myers (1977), when a firm has an excessive amount of debt, the equityholders tend to forgo a new project even if the project has a positive net present value (NPV). The reason is that existing creditors have priority over the payoffs from new investments, but equityholders have to bear the whole investment costs. As such, the government appears to be able to mitigate this agency friction between equity and debt by lightening the investment costs borne by equityholders. This simple observation indeed holds true in the original model developed by Myers (1977). In other words, in such a standard setup, reducing the investment costs increases a firm value by restoring equityholders' incentives to undertake positivie-NPV projects.

This common wisdom, however, looks inconsistent with our new empirical findings plotted in Figure 1. This graph shows that the average investment rate of non-investment grade firms is negatively related with the average recovery rate of defaulted corporate bonds. More specifically, over the period from 1984 to 2016, a 1% increase in the investment rate by non-investment grade firms predicts a 1.5% decrease in the average bond recovery rate two years later.[1] This new empirical pattern suggests that certain changes in government policies or economic conditions that boost more investments do not necessarily lead to an improvement in a debt value or even a firm value.

One economic channel that can explain this empirical pattern is inefficient overinvestments, which is generally caused by either the risk-shifting problem or managers' overcon-

[1]We introduce a two-year gap between the investment rate and the recovery rate, because new investment in tangible capital generally takes more than one year to be complete. Thus, a one-year gap looks a bit short to examine a causal relationship between ex-ante investments and the ex-post recovery rate. Nonetheless, even if we replace the two-year gap by a one-year gap, those two variables still exhibit a statistically significant negative relationship, although R^2 is slightly reduced.

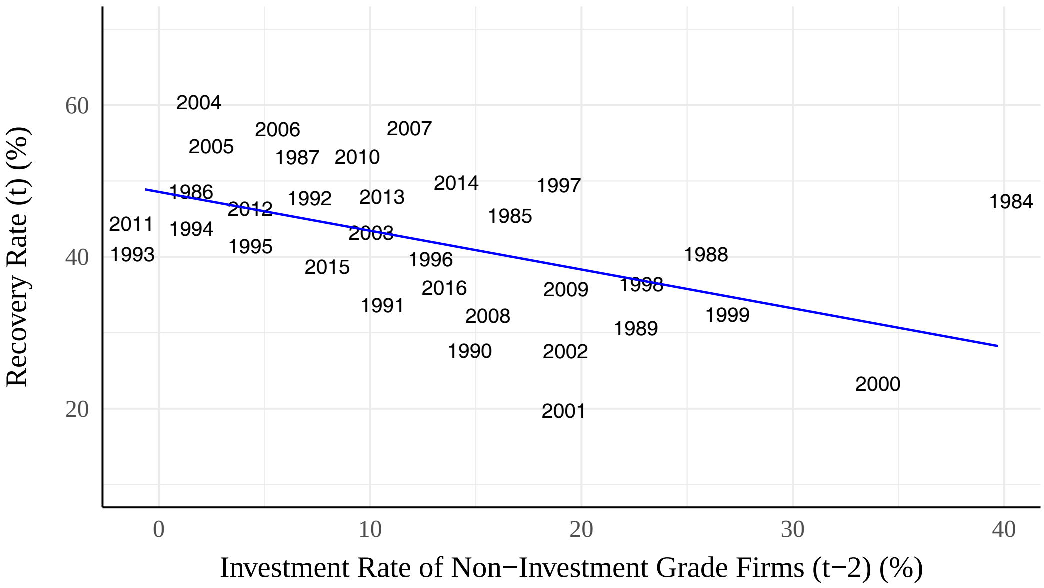

Figure 1: This figure plots the relationship between the issuer-weighted average investment rate of non-investment grade firms at year $t-2$ and the issuer-weighted average recovery rate of corporate bonds at year t, where t ranges from 1984 to 2016. The investment rate is defined as the total net investments in tangible capital such as plants, property, and equipment divided by the total asset value. The data on the investments and recovery rates come from Compustat and Moody's Default and Recovery Database, respectively.

fidence. Regarding the risk-shifting problem, as argued by Jensen and Meckling (1976), equityholders with significant debt have an incentive to undertake a risky negative-NPV project. But when this project fails, the bond recovery rate will be even lower compared to the case where equityholders do not invest in that project. Such a risk transfer from equityholders to creditors can thus generate a negative relationship between investments and the bond recovery rate. We can understand the overconfidence channel in a similar way.

Unlike this a bit straightforward mechanism, we introduce a new mechanism to show that, in the presence of debt overhang, policies seeking to foster investments even in positive-NPV projects can hurt a firm value by reducing the recovery value of assets in default. That is, the government's attempts to resolve the underinvestment problem may instead decrease a firm value or even welfare of the economy. Or, from the perspective of positive analysis, we can also say that any changes in economic conditions inducing more investments in positive-NPV projects may rather reduce a firm value.

To this aim, we develop a credit-risk model with multiple firms, in which the quality of each firm's existing asset is not publicly observable. Specifically, equityholders and creditors can observe their firms' asset qualities, but potential buyers of assets in default in the secondary market cannot.[2] Also, each firm has a new investment opportunity in a positive-NPV project, where equityholders control the investment decision. Thus, a standard debt-overhang problem arises in this model.

Moreover, every firm is exposed to an aggregate shock that can trigger a simultaneous default of multiple firms with different asset qualities. We can broadly consider this aggregate shock as a profitability shock, productivity shock, or liquidity shock. Because of this simultaneous default of different firms with unknown asset qualities, a typical adverse selection problem emerges in the secondary market. That is, creditors of good firms may rationally choose to retain their failed assets, even though doing so is costly to them. Here, asset retention incurs some inefficiency costs to creditors, because they typically do not have enough skills in reorganizing failed assets. Nonetheless, due to information asymmetry, some creditors decide to restructure their assets by their own instead of immediately liquidating them, especially when the liquidation value of bad assets is sufficiently low.

In this setting, why can policies of lightening the investment costs be ineffective? Our key observation is that equityholders of different firms have different incentives to undertake new projects and therefore, the investment decisions of equityholders can reveal their firms' asset qualities. Note that equityholders of a firm with an inferior asset are less willing to undertake a new project, because a larger portion of the outputs from the new project will be accrued to the creditors. For the opposite reason, equityholders of a good firm have larger incentives to undertake a new project. In other words, the debt-overhang problem is less severe to equityholders of good firms. Thus, in some situations, if only good firms invest in the new projects, uninformed potential buyers can rationally distinguish between good firms and bad firms from their past investment decisions. This way, equityholders' different incentives to undertake new investment can unintentionally eliminate information asymmetry

[2]In practice, of course, potential buyers can also extract some information about a failed firm using its financial statements, per se. But financial statements are usually not very useful for predicting a defaulted firm's future profitability. Moreover, a bankruptcy court typically imposes a deadline on the auction process for firms in default. As a result, potential buyers face difficulty estimating the true value of those firms. In the empirical literature, Wittenberg-Moerman (2008), Han and Zhou (2014), and Kedia and Zhou (2014) find that equityholders and creditors indeed have superior information about their firms than outside investors.

in the bankruptcy market.

Given this observation, suppose the government attempts to reduce the investment costs to encourage even bad firms to invest in positive-NPV projects. This policy certainly increases welfare to some extent by stimulating more investments in profitable projects. However, because all existing firms now invest in the new projects, their investment decisions do not provide any useful information about their assets. Therefore, information asymmetry emerges again in the secondary market. In other words, the above policy may hurt welfare by blurring the informational contents of investments. When this negative effect dominates the above positive effect, such a policy will indeed decrease welfare. This outcome generally occurs when the above aggregate shock is more likely to arrive or creditors are much less skilled in restructuring failed assets.

Now another important question naturally arises: Between our mechanism and the overinvestment mechanism, which mechanism mainly drives our motivating empirical fact plotted in Figure 1? To tackle this question, we examine the relationship between the ex-ante investment rate and the ex-post recovery rate in a more sophisticated way. We then provide some suggestive empirical evidence that supports our model rather than the overinvestment channel.

Specifically, note that the overinvestment channel predicts the negative relationship between the investment rate and the recovery rate at an individual firm level. For instance, remind that the risk-shifting phenomenon says that when a levered firm invests in a risky inefficient project, the bond recovery rate of this firm itself will be further lowered if a negative outcome is realized. In our model, however, when relatively bad firms increase their investments, the bond recovery rate of relatively good firms will be decreased at the simultaneous default event. Put differently, our model predicts that the recovery rate of good firms is negatively related with the investment rate of bad firms. We use this key distinction to study which channel better explains our motivating empirical fact.

Briefly, Table 2 shows that the bond recovery rate of highly-rated firms and the investment rate of low-rated firms have a statistically significant negative relationship. But the recovery rate and the investment rate do not exhibit any significant relationship within equally-rated firms. We can therefore argue that this additional empirical finding is more consistent with our model rather than the overinvestment channel. We discuss more details in Section 4.

Our paper contributes to the literature as follows. After Myers (1977) pointed out the debt overhang problem, many researchers have shown that debt overhang is a first-order friction that hinders both economic growth and economic recovery from recessions. For instance, Mello and Parsons (1992), Hennessy (2004), Hennessy et al. (2007), Moyen (2007), and Chen and Manso (2017) find that the agency cost of debt overhang is quantitatively significant.

Given the importance of this issue, researchers have studied how to resolve the debt overhang problem. Stulz and Johnson (1985) show that secured debt can alleviate the under-investment problem. Hackbarth and Mauer (2012) find that jointly optimal capital and debt priority structure can mitigate the conflicts between equityholders and creditors. Sundaresan et al. (2015) also analyze how debt priority structure affects the incentives for investment. Gertner and Scharfstein (1991), Titman and Tsyplakov (2007), and Diamond and He (2014) find some negative effects of short-term debt on investment. He (2011) analyzes the relationship between moral hazard and debt overhang. However, none of these papers argue that the underinvestment problem plays some positive role in the economy. Our paper shows that the presence of underinvestment can mitigate information asymmetry in the secondary market. Using this key insight, we show that reducing the investment cost may hurt firm value as well as the recovery value of assets in default.

The paper is organized as follows. Section 2 develops a simple model to illustrate the main idea. Section 3 develops a more sophisticated model. Section 4 provides some empirical evidence. Section 5 concludes. All technical arguments are included in the appendix.

2 Simple Model

2.1 Setup

The economy consists of a continuum of firms of measure one. There are only three dates, indexed by $t \in \{0, 1, 2\}$. Each firm operates an existing asset and owes some amount of debt.

Each firm also has an investment opportunity in a new project. The economy has a secondary market as well, in which assets in default are liquidated to potential asset buyers. All market participants, namely, equityholders, creditors, and potential buyers, are risk neutral and have a zero discount rate.

Each firm's existing asset will produce outputs at date 2. The amount of outputs depends on the firm's asset quality, which is predetermined in the beginning of date 0. Specifically, every asset can be either good or bad. A good asset will produce x_g and a bad asset will produce $x_b < x_g$, where both x_b and x_g are constants. But the asset quality $i \in \{b, g\}$ is known only to the firm's equityholders and creditors, but not to potential buyers in the secondary market.[3] A fraction π of the firms have good assets and the remaining firms have bad assets. We hereafter use the terms quality and type interchangeably.

As mentioned above, each firm has an investment opportunity that requires upfront costs of κ at date 0. This new project will produce outputs of y with certainty at date 2. As usual, equityholders control the investment and bear the whole investment costs. The investment decision cannot be delayed to date 1. Each firm's investment decision is publicly observable; that is, all market participants can see whether any given firm has invested in the new project.

Every firm is exposed to an aggregate shock at date 1. For ease of exposition, we model this shock as a profitability shock that affects the output price of the assets. Specifically, the output price is initially normalized to 1. But once the shock hits the economy, the price drops to $\gamma \in (0, 1)$; otherwise, the price remains at 1. This profitability shock arrives with probability $p \in [0, 1]$ and is publicly observable.

In this setting, consider an unlevered firm of type $i \in \{b, g\}$ for the moment. The date-0 value of the firm is given by

$$\max\{\underbrace{-\kappa + (p\gamma + 1 - p)(x_i + y)}_{\text{invests}}, \underbrace{(p\gamma + 1 - p)x_i}_{\text{does not invest}}\}. \tag{1}$$

That is, if the firm invests in the new project, the firm's expected profits will be $-\kappa + (p\gamma +$

[3] In practice, equityholders may have better information than creditors. But we do not pay attention to this additional friction, because what essentially matters in our model is that creditors have more precise information than outside potential buyers.

$1-p)(x_i+y)$; otherwise, the firm is expected to earn $(p\gamma+1-p)x_i$. Hence, the firm invests in the new project if and only if the project has a positive NPV, that is, $\kappa < (p\gamma+1-p)y$. Throughout this model, we assume this condition.

As well known, however, equityholders of a levered firm may forgo this positive-NPV project. Specifically, each firm in this model is required to repay a coupon c at date 1 and a face value F at date 2. Assuming equityholders have deep pockets, they can service the coupon payments at date 1, although their assets do not produce any cash flows at that date. Yet, due to limited liability, equityholders can strategically default at date 1, whenever they do not have an incentive to run their assets until date 2. When a firm defaults at date 1, the entire ownership of its assets is transferred to the creditors and therefore, the equityholders receive nothing upon default. Thus, equityholders choose to default at date 1 if and only if the continuation value of equity is below zero.

To simplify an equityholder's default decision, we focus on the following parameter conditions:

$$\gamma(x_g+y) < c+F < x_g \quad \text{and} \quad x_b < c+F < x_b+y. \tag{2}$$

The inequality $\gamma(x_g+y) < c+F$ means that when the aggregate shock hits the economy, both good firms and bad firms default regardless of whether they have invested in the new projects at date 0. That is, the aggregate shock is severe enough, thereby triggering all the firms to default. The inequality $c+F < x_g$ means that as long as the aggregate shock does not hit the economy, good firms never default regardless of whether they have invested at date 0. However, the inequality $x_b < c+F < x_b+y$ means that even if the aggregate shock does not hit the economy, bad firms still default if they have not invested at date 0. In other words, equityholders of a bad firm do not have enough incentives to meet the debt obligation unless they have launched the new project at date 0, even in the absence of the aggregate shock. For clarification, if we do not impose the conditions in (2), the economy will behave either a straightforward or an uninteresting way.

When a firm defaults at date 1, its creditors take over the firm's entire assets as mentioned above. The creditors can then either retain their assets or liquidate them to potential buyers immediately, where the first scenario specially means that the creditors restructure their assets by their own. But we assume that potential buyers are more skillful in restructuring failed assets than creditors. This assumption makes sense because creditors typically do not have enough experience in making any crucial business decisions in practice. Specifically, creditors in this model have a productivity of $\alpha \in (0, 1)$ and potential buyers have a full productivity of 1. That is, outputs will be reduced proportionally by a fraction $1 - \alpha$ if creditors manage their assets by themselves, whereas potential buyers do not incur any efficiency losses. In reality, potential buyers also tend to be less skillful than incumbent managers, but we do not consider this additional friction. What matters in our model is that potential buyers are at least more skillful than creditors.

Because of this difference in productivities, in the frictionless economy, creditors of failed firms must prefer to liquidate their assets to potential buyers to realize the gains from trade. However, in the presence of information asymmetry, creditors may choose not to liquidate their assets, especially when the liquidation value of bad assets is sufficiently low. More specifically, in this situation, creditors of failed firms play a game with potential buyers to decide whether to retain their assets or liquidate them, taking as given the potential buyers' beliefs about the qualities of liquidated assets. In equilibrium, potential buyers must correctly believe the qualities of those assets. For clarification, information asymmetry matters in the secondary market only when the aggregate shock hits the economy. The reason is that when the aggregate shock does not hit the economy, good firms never default and potential buyers can rationally infer this fact.

Most importantly, information asymmetry can be eliminated even when the aggregate shock hits the economy and triggers a simultaneous default of all existing firms. To see why, note that equityholders of an indebted firm may not always undertake the above positive-NPV project. Furthermore, equityholders of bad firms typically have even lower incentives to undertake the new projects than equityholders of good firms. We will analyze an equityholder's investment decision more precisely in the next section. As such, in cases where only good firms invest, potential buyers in the bankruptcy market can distinguish between good firms and bad firms from their past investment decisions. As a result, creditors of good firms can liquidate their assets at the price of $\gamma(x_g + y)$, whereas creditors of bad firms can liquidate their assets at the price of γx_b. This way, equityholders' different incentives to undertake the new projects can eliminate information asymmetry. Meanwhile, when every firm invests or no firms invest, information asymmetry indeed matters when the aggregate shock hits the economy.

2.2 Model Solutions

This section characterizes an equilibrium. We first solve an equityholder's problem and then pin down an equilibrium in the secondary market.

2.2.1 Equityholder's Problem

Recall that under the conditions in (2), every firm defaults at date 1 if the aggregate shock hits the economy. When the aggregate shock does not hit the economy, good firms never default, but bad firms still default unless they have launched the new projects. Thus, the equity values of a good firm and a bad firm at date 0 are respectively given by

$$E_g = \max\{\underbrace{-\kappa + (1-p)(x_g + y - c - F)}_{\text{invests}}, \underbrace{(1-p)(x_g - c - F)}_{\text{does not invest}}\} \tag{3}$$

and

$$E_b = \max\{\underbrace{-\kappa + (1-p)(x_b + y - c - F)}_{\text{invests}}, \underbrace{0}_{\text{does not invest}}\}. \tag{4}$$

We can thus deduce that a good firm invests if and only if

$$\kappa < (1-p)y, \tag{5}$$

whereas a bad firm invests if and only if

$$\kappa < (1-p)(x_b + y - c - F). \tag{6}$$

The first result implies that even a good firm forgos the positive-NPV project when $(1-p)y < \kappa < (p\gamma + 1 - p)y$. The second result implies that equityholders of a bad firm have even less incentives to undertake the new project, because $x_b - c - F < 0$. Intuitively, when the existing asset is expected to produce lower outputs, equityholders will benefit less from the new project. In other words, the debt-overhang problem is more severe to bad firms.

2.2.2 Equilibrium in the Secondary Market

This section pins down an equilibrium in the secondary market. As discussed above, we can focus on the case where the aggregate shock hits the economy, because otherwise, good firms never default. We consider the following three cases separately: (i) $\kappa < (1-p)(x_b+y-c-F)$, (ii) $(1-p)(x_b+y-c-F) < \kappa < (1-p)y$, and $(1-p)y < \kappa < (\gamma p + 1 - p)y$.

Case 1: $\kappa < (1-p)(x_b+y-c-F)$. In this case, every firm invests because the investment cost is sufficiently low. Thus, potential buyers cannot distinguish between good firms and bad firms based on their past investment decisions. Creditors therefore need to decide whether to retain or liquidate their assets as described above. As commonly known, two types of equilibria can arise in this case: a separating equilibrium and a pooling equilibrium. In a separating equilibrium, creditors of good firms restructure their assets by their own, whereas creditors of bad firms liquidate their assets. In a pooling equilibrium, all creditors liquidate their assets at the pooling price of the assets. But note that in a pooling equilibrium, no efficiency losses incur because all failed assets can be at least transferred to potential buyers. In this regard, we hereafter focus on a separating equilibrium by imposing some parameter restrictions, if needed, because our main interest lies in the welfare effects of government intervention policies.

Specifically, a separating equilibrium obtains when

$$x_b + y < \alpha(x_g + y), \tag{7}$$

in which creditors of bad firms liquidate their assets at the price of $\gamma(x_b + y)$ and creditors of good firms will earn $\gamma\alpha(x_g + y)$ by keeping their assets. To justify this outcome is indeed an equilibrium, notice that creditors of good firms have no incentives to mimic the creditors of bad firms, because $\gamma(x_b + y) < \gamma\alpha(x_g + y)$. That is, the liquidation value of bad assets is too low and therefore, the creditors of good firms rationally choose to retain their assets. Also, creditors of bad firms are not willing to retain their assets, because that strategy is always the worst option for the creditors of bad firms. Hence, the above equilibrium can indeed exist.

In this equilibrium, the date-0 debt values of a good firm and a bad firm are respectively given by

$$D_g = \underbrace{p\gamma\alpha(x_g + y)}_{\text{retention}} + \underbrace{(1-p)(c+F)}_{\text{fully repaid}} \quad \text{and} \quad D_b = \underbrace{p\gamma(x_b + y)}_{\text{liquidation}} + \underbrace{(1-p)(c+F)}_{\text{fully repaid}}.$$

The first term in D_g (resp. D_b) indicates that creditors of good (resp. bad) firms retain (resp. liquidate) their assets upon the arrival of the aggregate shock. The second terms in D_g and D_b indicate that creditors of both good firms and bad firms are fully repaid when the aggregate shock does not hit the economy. Compared to the economy without information asymmetry, creditors of good firms are worse off, because they now reorganize their assets by themselves instead of liquidating those assets. Meanwhile, information asymmetry does not affect the creditors of bad firms, because they can liquidate their assets at the fair price even in the presence of information asymmetry.

Case 2: $(1-p)(x_b + y - c - F) < \kappa < (1-p)y$. In this case, only good firms choose to invest at date 0, because the investment cost is moderate. Potential buyers can thus differentiate good firms from bad firms based on their past investment decisions. As a result, creditors of every individual firm can liquidate their assets at the fair price. The date-0 debt values of a good firm and a bad firm are thus respectively given by

$$D_g = \underbrace{p\gamma(x_g + y)}_{\text{liquidation}} + \underbrace{(1-p)(c+F)}_{\text{fully repaid}} \quad \text{and} \quad D_b = \underbrace{p\gamma x_b}_{\text{liquidation}} + \overbrace{(1-p)x_b}^{\text{liquidation}}.$$

The first terms in D_g and D_b indicate that creditors of both good firms and bad firms can liquidate their assets at the fair prices upon the arrival of the aggregate shock. The second term in D_g means that creditors of good firms are fully repaid if the aggregate shock does not hit the economy. Meanwhile, the second term in D_b means that even if the aggregate shock does not hit the economy, bad firms still default because they have not invested in the new projects. But, as mentioned above, since potential buyers can rationally infer this outcome, asymmetric information does not matter when only bad firms default. Thus, the liquidation value is simply given by x_b.

Case 3: $(1-p)y < \kappa < (\gamma p + 1 - p)y$. In this case, no firms invest in the new projects, because κ is sufficiently large. Therefore, the secondary market actually behaves almost the same way as in the first case. That is, a separating equilibrium obtains as long as $x_b < \alpha x_g$, in which creditors of only bad firms liquidate their assets. As such, the date-0 debt values of a good firm and a bad firm are respectively given by

$$D_g = \underbrace{p\gamma\alpha x_g}_{\text{retention}} + \underbrace{(1-p)(c+F)}_{\text{fully repaid}} \quad \text{and} \quad D_b = \underbrace{p\gamma x_b}_{\text{liquidation}} + \overbrace{(1-p)x_b}^{\text{liquidation}},$$

which can be similarly understood as in the above.

2.3 Main Results

This section discusses the main results of the model. We first examine the effects of the investment costs and then discuss the effects of other factors as well.

2.3.1 Effects of the Investment Costs

We can illustrate the key result of the paper as follows. Suppose κ is reduced from κ_1 to κ_2, where

$$(1-p)(x_b + y - c - F) < \kappa_1 < (1-p)y \quad \text{and} \quad \kappa_2 = 0. \tag{8}$$

When κ_1 satisfies this condition, we have seen that only good firms choose to invest. When the investment cost is 0, every firm certainly invests in the new project. Therefore, such a reduction in κ changes the equity values and debt values as follows:

$$\begin{aligned} \Delta E_g &= \underbrace{\kappa_1}_{\text{NPV effect}} > 0, \quad \Delta E_b = \underbrace{(1-p)(x_b + y - c - F)}_{\text{NPV effect}} > 0, \\ \Delta D_g &= \underbrace{-p\gamma(1-\alpha)(x_g + y)}_{\text{adverse selection effect}} < 0, \quad \Delta D_b = \underbrace{p\gamma y + (1-p)(c + F - x_b)}_{\text{NPV effect}} > 0. \end{aligned} \tag{9}$$

Note that the equity value of a good firm increases by κ_1, because equityholders can now undertake the new project for free. The equity value of a bad firm increases by $(1-p)(x_b + y - c - F)$, because even bad firms now invest in the new projects. The debt value of a bad firm increases by $p\gamma y + (1-p)(c + F - x_b)$, because (i) the new project launched at date 0 increases the liquidation value of a bad firm by γy and (ii) bad firms now do not default as long as the aggregate shock does not hit the economy. We can interpret all of those effects as positive-NPV effects. Meanwhile, the debt value of a good firm decreases by $p\gamma(1-\alpha)(x_g + y)$, because after the reduction in the investment costs, equityholders' investment decisions become uninformative and thereby the adverse selection problem emerges again. As such, we can say that reducing the investment costs does not necessarily benefit all the claim holders as in the original model by Myers (1977).

This adverse selection effect can actually dominate the other positive-NPV effects, so that the above policy can hurt even a firm value, especially for good firms. Given that the firm value is the sum of the equity value and the debt value, when κ decreases from κ_1 to κ_2, the present value of a good firm changes by

$$\Delta F_g := \Delta E_g + \Delta D_g = \kappa_1 - p\gamma(1-\alpha)(x_g + y). \tag{10}$$

This expression means that reducing the investment costs is likely to harm the firm value when p is large, γ is high, α is low, or $x_g + y$ is large. But when γ or $x_g + y$ is too large, the parameter conditions in (2) will be violated. Thus, we here focus on the effects of the parameters p and α. Then the above result indicates that when the aggregate shock is more likely to arrive or creditors have poorer skills in restructuring their assets, reducing κ can even lower the present value of good firms. This result is intuitive enough because in such cases, creditors of good firms are more concerned about the adverse selection problem that may occur at date 1. Lastly, note that κ_1 does not need to be equal to zero. As long as $\kappa_2 < (1-p)(x_b + y - c - F)$, we can derive a similar result.

Now we consider the other case in which κ is reduced from κ_1 to κ_2, where

$$(1-p)y < \kappa_1 < (\gamma p + 1 - p)y \quad \text{and} \quad (1-p)(x_b + y - c - F) < \kappa_2 < (1-p)y. \tag{11}$$

In this case, we can easily see that all the claim holders are better off, because such a change now eliminates the adverse selection problem rather than triggering this problem as in the previous case. Together with the above positive-NPV effects, this policy change certainly benefits the whole economy. We omit the details.

2.3.2 Effects of Other Factors

We have thus far focused on the role of the investment costs to derive somewhat counterintuitive result. But we need not restrict our attention to this factor only, because any changes in other factors that induce more investments can decrease a debt value or a firm value through the same mechanism. Specifically, as in the previous section, suppose our economy currently satisfies the condition $(1-p)(x_b + y - c - F) < \kappa < (1-p)y$. But imagine that some unexpected shock suddenly hits the economy and then twists the market condition to another one such that $\kappa < (1-p)(x_b + y - c - F)$. For instance, we can think of this shock as a sudden decrease in the probability that the aggregate profitability shock hits the economy next year. Once this unexpected shock occurs, we know that even bad firms invest in the new projects. As a result, equityholders' investment decisions become less informative, which can particularly harm the creditors of good firms. In this regard, even when the aggregate profitability shock becomes less threatening, such a change does not guarantee improvements in a debt value or a firm value. We can understand other parameters such as x_b, y, c, and F in a similar manner.

3 Dynamic Model

We now build a more sophisticated model by extending Leland (1994), which can be calibrated to data more easily. The main difference from the simple model is that a firm's asset quality fluctuates over time and creditors use partial asset retention as the signaling device. Time is continuous and runs from 0 to ∞. There is a continuum of ex-ante identical firms, each indexed by $i \in [0, 1]$. But, to ease the exposition, we will mainly consider only one firm unless otherwise stated. All market participants are risk neutral and have a discount rate r. Every firm has a representative equityholder and a representative creditor.

3.1 Setup

3.1.1 Firm Asset and Investment

The firm manages an asset in place that produces after-tax cash flows $A_t x_t dt$ over each time interval $[t, t+dt)$. Here, A_t denotes the aggregate level of profitability at time t, such as the output price, and x_t denotes the firm's asset quality at time t. As in the simple model, the profitability A_t is publicly observable, but the asset quality x_t is privately observable only to the firm's equityholder and creditor. Both A_t and x_t vary over time.

The profitability A_t is initially equal to 1, but drops to $\gamma < 1$ if an aggregate profitability shock hits the economy. This shock arrives with Poisson intensity ϕ. For simplicity, we do not consider a positive profitability shock that pushes back the profitability to 1. We call the periods in which the profitability is equal to 1 (resp. γ) the good (resp. bad) time.

The asset quality x_t evolves as

$$\frac{dx_t}{x_t} = \mu_t dt + \sigma dZ_t,$$

where μ_t is the growth rate at time t, $\sigma > 0$ is the volatility, and Z_t is a standard idiosyncratic Brownian motion. The growth rate μ_t is determined by the firm's investment decision, which is controlled by the equityholder. Specifically, as in Diamond and He (2014), the equityholder can increase the growth rate by investing in an additional project at the flow costs of $\kappa x_t dt$, where $0 \leq \kappa \leq 1$. That is, the growth rate stays at μ_L over the periods in which the equityholder does not invest in the project. But the growth rate stays at μ_H over the periods in which the firm invests in the project, where $\mu_L < \mu_H$. In addition, this investment project is actually available to the firm only during the good time. We can easily relax this assumption, but doing so will not change our main result qualitatively. Lastly, we assume that the investment cost κ is small enough so that the above investment has positive NPV regardless of the current asset quality. Thus, in case where the firm has no leverage, it will invest in the project at every time. We will provide a specific parameter condition for κ shortly.

In this circumstance, the unlevered firm value at time t is given by

$$F^g(x_t) = \frac{1-\kappa+\frac{\phi\gamma}{r-\mu_L}}{r+\phi-\mu_H}x_t \quad \text{and} \quad F^b(x_t) = \frac{\gamma}{r-\mu_L}x_t,$$

during the good time and the bad time, respectively. Here, we assume $r+\phi > \mu_H$ and $r > \mu_L$ to ensure that the unlevered firm value is always positive. Further, to make sure the above investment has positive NPV, we assume that

$$\kappa(r-\mu_L)(r+\phi-\mu_L) < (\mu_H-\mu_L)(r-\mu_L+\phi\gamma). \tag{12}$$

This condition actually comes from $\kappa x_t < (\mu_H-\mu_L)F^g(x_t)$, which means that the investment cost is lower than the investment benefit to the unlevered firm.

However, the equityholder of a levered firm may not want to invest in the project because of the classical debt overhang problem. That is, since the creditor has the higher priority on the payoffs from the investment, the equityholder may not have enough incentives to bear the investment costs, especially when the firm is expected to default soon. In this regard, we can postulate that the equityholder invests in the project only when $x_t \geq x_I$ for some threshold x_I, which will be determined endogenously later. Importantly, as in the static model, the firm's investment decision $1_{x_t \geq x_I}$ is publicly observable. Hence, at the default date, even the potential buyers can see the firm's most recent investment decision.

For clarification, the potential buyers cannot directly observe the investment costs incurred, that is, $\kappa x_t 1_{x_t \geq x_I}$. This assumption rules out the case where the potential buyers can perfectly back out the asset quality x_t from the investments. The rationale for this assumption is that, in practice, estimating the amount of past investments precisely is challenging, although figuring out whether a firm has recently made sizable investments is relatively easy. This argument thus justifies our assumption to some extent. In fact, to avoid this subtle issue, we can actually assume that the investment project requires a constant flow cost κdt instead of the variable flow cost $\kappa x_t dt$. But we will not adopt this assumption because the latter setup is more commonly used in the literature.

3.1.2 Firm Liability

The firm has some amount of debt outstanding. That is, as in Leland (1994), the firm issues one unit of perpetuity debt that pays coupon c at every time. Let π denote the corporate tax rate. Then, the after-tax net cash flow to the equityholder is given by

$$\begin{cases} x_t - (1-\pi)c - \kappa x_t 1_{x_t \geq x_I} & \text{in the good time} \\ \gamma x_t - (1-\pi)c & \text{in the bad time.} \end{cases}$$

Here, the equityholder has a deep pocket. Thus, even when the net cash flow falls below 0, the equityholder can inject more money into the firm as long as the equity value is positive. But when the equity value hits zero, the equityholder decides to default on the debt payment. In this regard, we can consider that the equityholder defaults when x_t hits x_D^g (resp. x_D^b) during the good (resp. bad) time for some thresholds x_D^g and x_D^b, to be determined endogenously later.

Importantly, there is another default scenario. That is, when the profitability shock hits the economy, the firm defaults immediately if its asset quality x_t lies below x_D^b at that time. In fact, the asymmetric information problem will arise only in this default scenario, which I will discuss shortly.

3.1.3 Asset Liquidation

Once the firm defaults, the creditor takes over the entire ownership over the firm's asset. But the creditor is a less skilled asset manager than the potential buyers. Specifically, the creditor can extract only a fraction α of the cash flows from the asset, while the potential buyers can extract a fraction β of the cash flows from the same asset, where $0 \leq \alpha \leq \beta \leq 1$. As a result, the time-$t$ present value of the failed asset to the creditor is equal to $\alpha F^g(x_t)$ and $\alpha F^b(x_t)$ during the good time and the bad time, respectively. Similarly, the potential buyers value the asset as $\beta F^g(x_t)$ and $\beta F^b(x_t)$ during the good time and the bad time, respectively. From now, let us use $H^g = \frac{F^g(x)}{x}$ and $H^b = \frac{F^b(x)}{x}$.

Now, consider three different default scenarios described above. In the first case, the firm defaults when its asset quality hits x_D^g during the good time. Thus, the potential buyers can rationally infer the firm's asset quality precisely because the aggregate state of profitability is publicly observable. Thus, the creditor can liquidate her asset at the price of $\beta H^g x_D^g$. In the second case where the firm defaults when its asset quality hits x_D^b during the bad time, the creditor can liquidate her asset at the price of $\beta H^b x_D^b$ by the same reason. However, in the third case where the firm defaults upon the arrival of the profitability shock, the potential buyers cannot infer the firm's asset quality precisely. The reason is that the asset quality x_t can be any number between x_D^g and x_D^b. That is, recall that there is a continuum of firms whose asset qualities are exposed to idiosyncratic shocks. Thus, when the profitability shock hits the economy, the firms whose asset qualities belong to $[x_D^g, x_D^b]$ default simultaneously. As a result, the creditor of some failed firm may not want to liquidate her asset, especially when her asset quality is relatively high.

In this model, following Leland and Pyle (1977), we allow the creditor to retain a partial fraction of her asset to signal the asset's true quality. Of course, partial asset retention is costly to the creditor because the potential buyers are more skillful asset users. But the creditor is willing to bear the costs to reveal the true asset quality.

Importantly, the creditor does not need to differentiate herself from all other creditors. The reason is that the potential buyers can infer at least whether the firm's asset quality is larger than x_I or not from the firm's most recent investment decision. In particular, consider the case where $x_D^g < x_I < x_D^b$. From now, we call the creditor, whose asset quality is equal to x, the creditor of type x. Then, the above argument says that the creditor of type $x \in [x_D^g, x_I)$ does not need to differentiate herself from the creditor of type y, if $y \in [x_I, x_D^b]$. However, the creditor of type x still needs to distinguish herself from the creditor of type z, if $z \in [x_D^g, x_I)$. We can similarly understand the behavior of a creditor whose type belongs to $[x_I, x_D^b]$.

Meanwhile, in the other case where $x_D^g < x_D^b < x_I$, the potential buyers cannot extract any information from a firm's recent investment decision. Thus, the creditors of failed firms play a signaling game all together when the profitability shock hits the economy.

3.2 Model Solutions

We now characterize an equilibrium. We first analyze the signaling game in the secondary market. We then pin down optimal default and investment thresholds.

3.2.1 Separating Equilibrium in the Secondary Market

This section characterizes a separating equilibrium in the secondary market for any given thresholds $\{x_D^g, x_D^b, x_I\}$. As mentioned above, we can focus on the third default scenario that is caused by the profitability shock. To begin with, suppose that the creditor of type $x \in [x_D^g, x_D^b]$ retains a fraction $f(x)$ of her asset to signal her type. The potential buyers also believe that such a creditor retains the fraction $f(x)$ of her asset. Then, consider the following two cases: (i) $x_D^g < x_I < x_D^b$ and (ii) $x_D^g < x_D^b < x_I$.

Case 1: In this case, the potential buyers can see whether a creditor's type belongs to $[x_D^g, x_I)$ or $[x_I, x_D^b]$. So, let us consider the creditors whose types belong to $[x_D^g, x_I)$ first. Suppose a creditor of type $x \in [x_D^g, x_I)$ mimics another creditor of type $y \in [x_D^g, x_I)$. Then, the former creditor is expected to earn $f(y)\alpha H^b x + (1 - f(y))\beta H^b y$ by retaining the fraction $f(y)$ of her asset and liquidating the remaining fraction of the asset. Regarding off-equilibrium beliefs, if the creditor of type $x \in [x_D^g, x_I)$ retains a fraction ξ of her asset, where there is no $y \in [x_D^g, x_I)$ such that $\xi = f(y)$, the potential buyers assign the worst type x_D^g to that creditor. Thus, she is expected to earn $\xi\alpha H^b x + (1 - \xi)\beta H^b x_D^g$.

In this setting, a creditor of type $x \in [x_D^g, x_I)$ solves the following problem:

$$\max_{y \in [x_D^g, x_I)} f(y)\alpha H^b x + (1 - f(y)\beta H^b y. \tag{13}$$

Here, we have intentionally ignored the off-equilibrium beliefs because no creditors will choose an off-equilibrium retention ratio in equilibrium. But we will verify this argument rigorously later. To solve the problem (13), note that every creditor truthfully reveals her asset quality in separating equilibrium. Thus, $f(x)$ must satisfy the following first-order condition (FOC):

$$\underbrace{(\beta - \alpha)xf'(x)}_{\text{costs}} = \underbrace{\beta(1 - f(x))}_{\text{benefits}}, \quad \forall x \in [x_D^g, x_I). \tag{14}$$

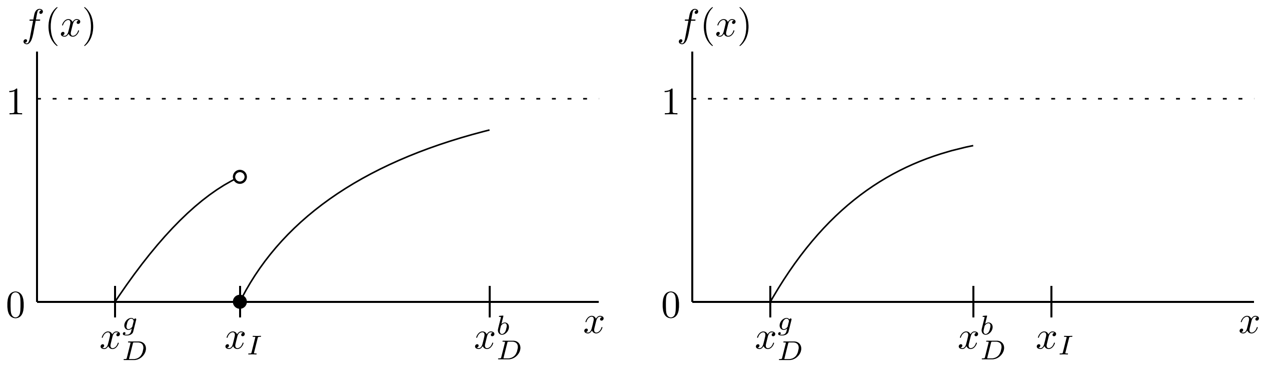

Figure 2: This figure plots the retention ratio $f(x)$. The left panel corresponds to the case where $x_D^g < x_I < x_D^b$. The right panel corresponds to the case where $x_D^g < x_D^b < x_I$.

This condition implies that $f'(x) \geq 0$, meaning that a creditor of a higher type retains a more fraction of her asset to distinguish herself from other creditors of lower types. Moreover, $f(x_D^g)$ must be 0 because otherwise the creditor of the worst type x_D^g can increase her profits by selling the entire fraction of her asset due to the above off-equilibrium beliefs. Using this boundary condition $f(x_D^g) = 0$, the solution to equation (14) is given by the first line in (15)

$$f(x) = \begin{cases} 1 - \left(\frac{x_D^g}{x}\right)^{\frac{\beta}{\beta-\alpha}}, & \forall x \in [x_D^g, x_I) \\ 1 - \left(\frac{x^I}{x}\right)^{\frac{\beta}{\beta-\alpha}}, & \forall x \in [x_I, x_D^b]. \end{cases} \tag{15}$$

In Appendix 6.1, we show that this formula for $f(x)$ indeed satisfies the global optimality condition by considering the off-equilibrium beliefs as well.

As expected, the second line in (15) corresponds to an optimal retention ratio chosen by a creditor of type $x \in [x_I, x_D^b)$ in equilibrium. We can derive this formula by the same method used above. The left panel in Figure 2 plots the retention ratio $f(x)$ for all $x \in [x_D^g, x_D^b]$. Note that $f(x)$ jumps down at x_I, although $f(x)$ increases in x over each of the subintervals, $[x_D^g, x_I)$ and $[x_I, x_D^b]$. This property will play a crucial role when we examine the informational role of investment.

Case 2: In the second case where $x_D^g < x_D^b < x_I$, the potential buyers cannot extract any information about a firm's asset quality from its recent investment decision. Thus, applying the above method to the entire interval $[x_D^g, x_D^b]$, we can derive that the retention ratio for a creditor of type $x \in [x_D^g, x_D^b]$ must be equal to

$$f(x) = 1 - \left(\frac{x_D^g}{x}\right)^{\frac{\beta}{\beta-\alpha}}, \quad \forall x \in [x_D^g, x_D^b]. \tag{16}$$

The right panel in Figure 2 plots the retention ratio in this case. Note that $f(x)$ keeps increasing over the whole interval $[x_D^g, x_D^b]$.

3.2.2 Equity Value

In this section, we compute the equity value together with optimal default and investment thresholds. Let $E^g(x)$ and $E^b(x)$ denote the arbitrage-free equity values during the good time and the bad time, respectively. Then, a standard continuous-time technique implies that $E^g(x)$ and $E^b(x)$ satisfy the following system of Hamilton-Jacobi-Bellman (HJB) equations:

$$\begin{cases} rE^g(x) = x - \kappa 1_{x\geq x_I} x - \tilde{c} + \phi(E^b(x) - E^g(x)) + (\mu_L + \delta 1_{x\geq x_I}) x E_x^g(x) + \frac{\sigma^2 x^2}{2} E_{xx}^g(x) \\ rE^b(x) = \gamma x - \tilde{c} + \mu_L x E_x^b(x) + \frac{\sigma^2 x^2}{2} E_{xx}^b(x), \end{cases}$$

subject to $E^g(x_D^g) = E_x^g(x_D^g) = E^b(x_D^b) = E_x^b(x_D^b) = 0$ and $E_x^g(x_I) = \frac{\kappa}{\delta}$, where $\tilde{c} = (1-\pi)c$ and $\delta = \mu_H - \mu_L$. In addition, $E^b(x) = 0$ for all $x \in [x_D^g, x_D^b]$ because the equityholder immediately defaults upon the arrival of the profitability shock.

Regarding the first equation, the left-hand side is the required return. The sum of the first three terms in the right-hand side is the after-tax net cash flow. The fourth term indicates the net change in the equity value due to the profitability shock. The remaining terms denote the effect of the fluctuations in the asset quality on the equity value. We can understand the second equation similarly.

The boundary conditions such as $E^g(x_D^g) = E_x^g(x_D^g) = E^b(x_D^b) = E_x^b(x_D^b) = 0$ correspond to the standard value-matching and smooth-pasting conditions for optimal default thresholds, x_D^g and x_D^b. The other condition $E_x^g(x_I) = \frac{\kappa}{\delta}$ indicates that the equityholder is indifferent between investing and not investing is the firm's asset quality equals x_I. In Appendix 6.2, we solve for equilibrium thresholds $\{x_D^g, x_D^b, x_I\}$ in almost closed form.

3.2.3 Debt Value

Now, we will compute the debt value. Let $D^g(x)$ and $D^b(x)$ denote the arbitrage-free debt values during the good time and the bad time, respectively. Then, a standard continuous-time technique again implies that

$$\begin{cases} rD^g(x) = c + \phi(D^b(x) - D^g(x)) + (\mu_L + \delta 1_{x \geq x_I})xD^g_x(x) + \frac{\sigma^2 x^2}{2} D^g_{xx}(x) \\ rD^b(x) = c + \mu_L x D^b_x(x) + \frac{\sigma^2 x^2}{2} D^b_{xx}(x), \end{cases} \tag{17}$$

subject to $D^g(x^g_D) = \beta H^g x^g_D$ and $D^b(x^b_D) = \beta H^b x^b_D$. In addition, $D^b(x)$ for $x \in [x^g_D, x^b_D)$ is given by

$$R(x) = f(x)\alpha H^b x + (1 - f(x))\beta H^b x,$$

where $f(x)$ is the equilibrium retention ratio obtained in Section 3.2.1. That is, $R(x)$ is the recovery value of a failed asset with quality x upon the arrival of the profitability shock.

Regarding the first equation, the left-hand side is the required return. The first term in the right-hand side is the coupon payment. The second term indicates the net change in the debt value due to the profitability shock. The remaining terms denote the effect of the fluctuations in the asset quality on the debt value. We can similarly understand the second equation.

3.3 Model Implications

We now discuss our model implications. We first analyze the informational role of investment. We then present comparative statics results with respect to numerous model parameters.

3.3.1 Informational Role of Investment

In this section, we examine how a change in the investment threshold x_I affects the asset retention ratio $f(x)$, by fixing the default thresholds x^g_D and x^b_D. By doing so, we can isolate the informational effect of investment on the secondary market liquidity.

The left panel of Figure 3 plots the asset retention ratio $f(x; x_I)$ under three different thresholds $x_I \in \{x^1_I, x^2_I, x^3_I\}$ such that $x^1_I < x^2_I < x^3_I$. Specifically, the solid, dotted, and

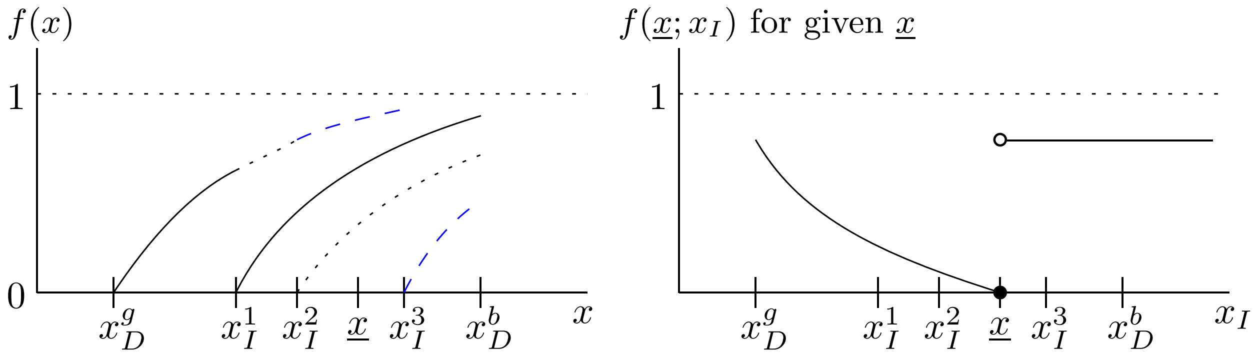

Figure 3: The left panel shows how the retention ratio $f(x)$ changes as the investment threshold increases from x_I^1 to x_I^3. The right panel plots the retention ratio $f(\underline{x}; x_I)$ as the function of x_I for a given $\underline{x}$.

dashed lines plot $f(x; x_I)$ for $x_I \in \{x_I^1, x_I^2, x_I^3\}$, respectively. We then pick another point $\underline{x}$ satisfying $x_I^2 < \underline{x} < x_I^3$ and plot $f(\underline{x}; x_I)$ as a function of x_I in the right panel of Figure 3.

First, when the investment threshold increases from x_I^1 to x_I^2, the retention ratio $f(\underline{x}; x_I^2)$ becomes lower than $f(\underline{x}; x_I^1)$. The reason is that when x_I changes from x_I^1 to x_I^2, the creditor of type $\underline{x}$ differentiates herself from the creditors whose types are above x_I^2 rather than above x_I^1. That is, the creditor of type $\underline{x}$ now belongs to a smaller pool of creditors. Thus, the retention ratio goes down. Applying the same argument, we can show that $f(\underline{x}; x_I)$ keeps decreasing in x_I until x_I reaches $\underline{x}$. In particular, when x_I equals $\underline{x}$, the retention ratio $f(\underline{x}; \underline{x})$ becomes 0. The decreasing curve in the right panel of Figure 3 illustrates these results.

Second, if the investment threshold further increases to x_I^3, the retention ratio $f(\underline{x}; x_I^3)$ becomes larger than $f(\underline{x}; x_I^1)$. The reason is that when x_I is equal to x_I^3, the firm with asset quality $\underline{x}$ does not invest and thus, the creditor of type $\underline{x}$ needs to differentiate herself from the creditors whose types are above x_D^g rather than above x_I^1. That is, the creditor of type $\underline{x}$ now belongs to a larger pool of creditors. Thus, the retention ratio goes up. The right panel of Figure 3 indeed shows that the retention ratio $f(\underline{x}; x_I)$ jumps up as soon as x_I exceeds $\underline{x}$. But $f(\underline{x}; x_I)$ remains constant when x_I increases further because the default threshold x_D^g is fixed here.

In sum, the retention ratio $f(\underline{x}; x_I)$ is non-monotone in x_I. When x_I decreases from x_I^3 to x_I^2, the retention ratio $f(\underline{x}; x_I)$ increases. But when x_I further decreases from x_I^2 to x_I^1, the retention ratio $f(\underline{x}; x_I)$ increases. We will use this crucial property to show that the effect of

Table 1: Baseline Parameters

Risk-free rate	$r = 7\%$
Corporate tax rate	$\pi = 27\%$
Productivity of the creditors	$\alpha = 40\%$
Productivity of the asset buyers	$\beta = 80\%$
Growth rate without investment	$\mu_L = 0\%$
Growth rate under investment	$\mu_H = 6\%$
Asset volatility	$\sigma = 15\%$
Arrival intensity for the profitability shock	$\phi = 2$
Coupon payment	$c = 10$

a change in the investment cost on firm value is also non-monotone.

3.3.2 Model Parameters

Before we present the comparative statics results, let us choose benchmark parameter values. Table 1 shows the benchmark parameter values. We choose $r = 7\%$, which is consistent with the interest rate used in Hackbarth et al. (2006), Huang and Huang (2012), and He and Xiong (2012). We choose the tax rate as $\pi = 27\%$. Specifically, using the marginal corporate tax rate 35%, the effective bond income tax rate 25%, and the marginal tax rate of 15% for capital gains, we can estimate the tax benefit of debt as $1 - \frac{(1-0.35)\times(1-0.15)}{1-0.25} = 26.5\%$ based on the formula in Miller (1977). We choose $\alpha = 40\%$ and $\beta = 80\%$ because the lowest and highest recovery rates are around 40% and 80%, respectively, according to Chen (2010). According to Miao (2005), the average growth rate and volatility of cash flows for the firms listed in Standard & Poor 500 are roughly 2.5% and 25%, respectively. So, we appropriately choose $\mu_L = 0\%$, $\mu_H = 6\%$, and $\sigma = 15\%$. We set $\phi = 2$, meaning that the profitability shock is expected to arrive in 6 months. We also appropriately choose $\kappa = 6\%$. Lastly, we normalize the coupon payment to 10.

3.3.3 Effects of the Investment Costs

We first examine how the investment cost affects the firms. At first glance, reducing the investment cost seems to obviously mitigate the debt overhang problem. But we will show that such a policy may benefit only equityholders, but not creditors.

The change in the investment cost basically has three effects. First, the change in κ directly affects the net cash flows to the firms. Second, the change in κ alters the equityholders' incentive to default. Third, the change in κ influences the equityholders' incentive to invest. We call these three channels the NPV channel, default channel, and information channel, respectively.

Specifically, suppose κ is decreased. Then, first, the NPV-channel certainly increases the equity value because the equityholders are required to pay less for investment when κ decreases. However, But the debt value is not directly affected by this channel because the creditors do not bear the investment cost.

Second, the default channel also pushes up the equity value because the equityholders choose to default later when the investment cost is lowered. But this channel can actually make the debt value either larger or smaller. On the one hand, the debt value can increase because the default risk is reduced when the default thresholds are lowered. On the other hand, when the default threshold in the good state, x_D^g, is lowered, the recovery value for the firms with asset quality $x \in [x_D^g, x_I)$ will decrease. This is because the asymmetric information problem for those firms deteriorates when x_D^g decreases. Thus, the debt value might be pushed down. Nonetheless, under most of parameter values, this negative effect tends to be dominated by the above positive effect of the delayed default.

Third, the information channel also increases the equity value because the asset growth rate is pushed up earlier when the equityholders invest earlier. But this channel may increase or decrease the debt value as intensively discussed above.

We now ask when does the information channel matter? More specifically, when does the negative effect of the information channel become strong enough so that the debt and firm values can go down when κ is lowered? To answer this question, first, note that the creditors of the firms with lower asset qualities are more sensitive to this channel, because those firms are more exposed to the profitability-shock driven default risk. Second, the adverse effect of the information channel is more problematic when the arrival intensity of the negative profitability shock, ϕ, is large. This is because when ϕ is small, the creditors in the good time are not concerned about the asset recovery value that much. In sum, while reducing the investment cost always increases the equity value, that policy may not raise the debt value

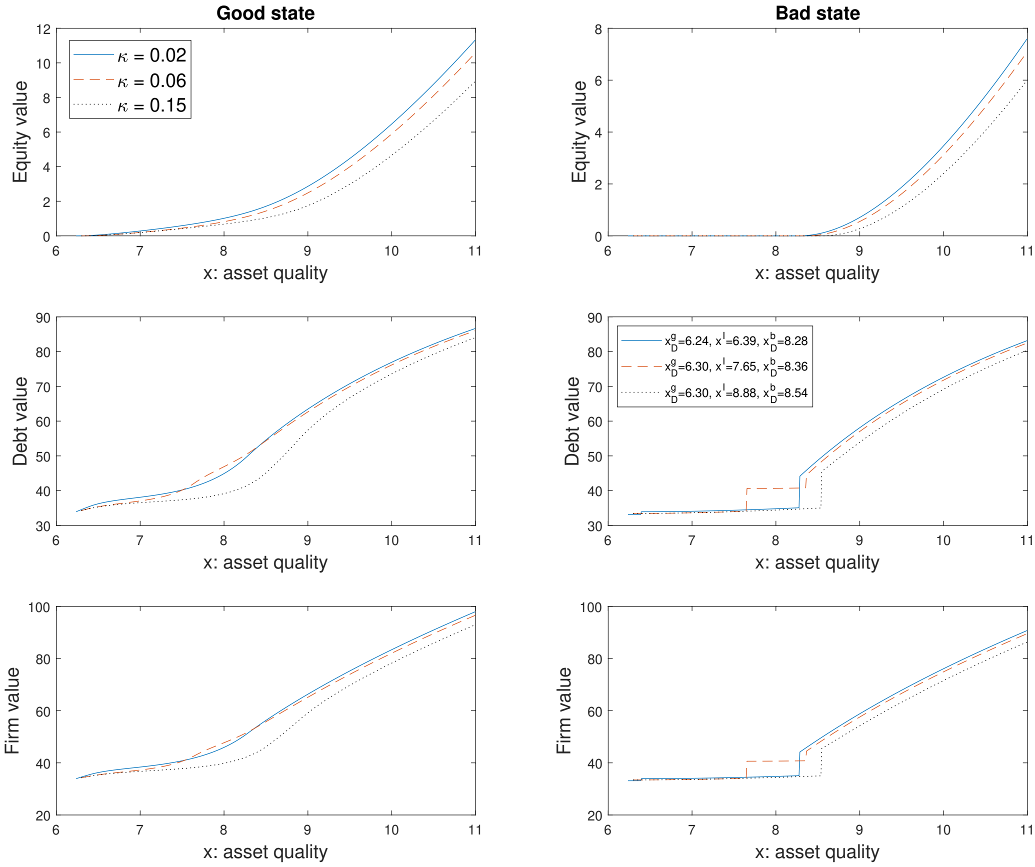

Figure 4: This figure plots the effect of a change in the investment cost κ. The upper, middle, lower panels plot the equity, debt, and firm values, respectively. The left and right panels correspond to the good time and the bad time, respectively.

mainly because of the information channel. Moreover, the magnitude of the informational effect will heavily depend on the arrival intensity of the negative profitability shock. In this regard, we will look at the effect of the change in κ under two different sets of the parameter values.

Figure 4 shows the effect of a change in the investment cost on the equity value, debt value, and firm value under the parameter values in Table 1.

From the upper two panels, we confirm that the equity value decreases in the investment cost in both states. To examine the effect on the debt value, we consider the following two cases: (i) when κ is reduced from 0.06 to 0.02 and (ii) when κ is reduced from 0.15 to 0.06.

In the first case, let us first look at the effect on the debt value in the bad state, which is shown at the middle-right panel of Figure 4. In that panel, recall that the debt value $D^b(x)$ for $x \in [x_D^g, x_D^b)$ corresponds to the recovery value $R(x)$ itself. Note that when $\kappa = 0.06$, we have $x_D^g = 6.30$, $x_I = 7.65$, and $x_D^b = 8.36$. But when $\kappa = 0.02$, we have $x_D^g = 6.24$, $x_I = 6.39$, and $x_D^b = 8.28$.

The figure shows that $D^b(x)$ increases for every $x \in [7.65, 8.28)$ as κ decreases from 0.06 to 0.02. This is because when κ is reduced in that way, the creditor of type $x \in [7.65, 8.28)$ needs to differentiate herself from the creditors of the type above 6.39 rather than above 7.65. Put differently, the retention ratio for the creditors of the type above the old investment threshold, $x_I = 7.65$, increases, because more firms choose to invest after the investment cost is cut down. Basically, this scenario corresponds to the case in which the investment threshold x_I^2 is lowered to x_I^1 in Figure 3.

Meanwhile, $D^b(x)$ decreases for every $x \in [6.39, 7.65)$ as κ decreases, although this pattern is not that visible in the figure. The reason is that when κ decreases from 0.06 to 0.02, the creditor of type $x \in [6.39, 7.65)$ only needs to differentiate herself from the creditors of the types above 6.39 rather than above 6.30. In other words, the firms with asset qualities in $[6.39, 7.65)$ now face less severe information asymmetry because the new investment threshold $x_I = 6.39$ lies above the old default threshold $x_D^g = 6.30$. This scenario corresponds to the case where the investment threshold x_I^3 is lowered to x_I^1 in Figure 3.

Importantly, we can find this non-monotone effect on the debt value even in the good time as shown in the middle-left panel in Figure 4. Specifically, recall that if the investment cost is reduced, the debt value can increase because the equityholders choose to default later. But the creditors in the good time are also concerned about the future recovery value for failed assets. When ϕ is particularly large, those creditors care about the recovery value more seriously. However, the recovery value may rather drop if the equityholders invest earlier. Thus, the debt value in the good state can drop as well, especially when ϕ is high.

We now consider the second case in which κ is reduced from 0.15 to 0.06. In contrast to the previous case, the debt value in this case increases for all x in both good and bad states, as in the two middle panels in Figure 4. To see why, note that when the investment cost is sufficiently large, the investment threshold lies above the default threshold in the bad state. Specifically, when $\kappa = 0.15$, we have $x_D^g = 6.30$, $x_D^b = 8.54$, and $x_I = 8.88$. As a result, the

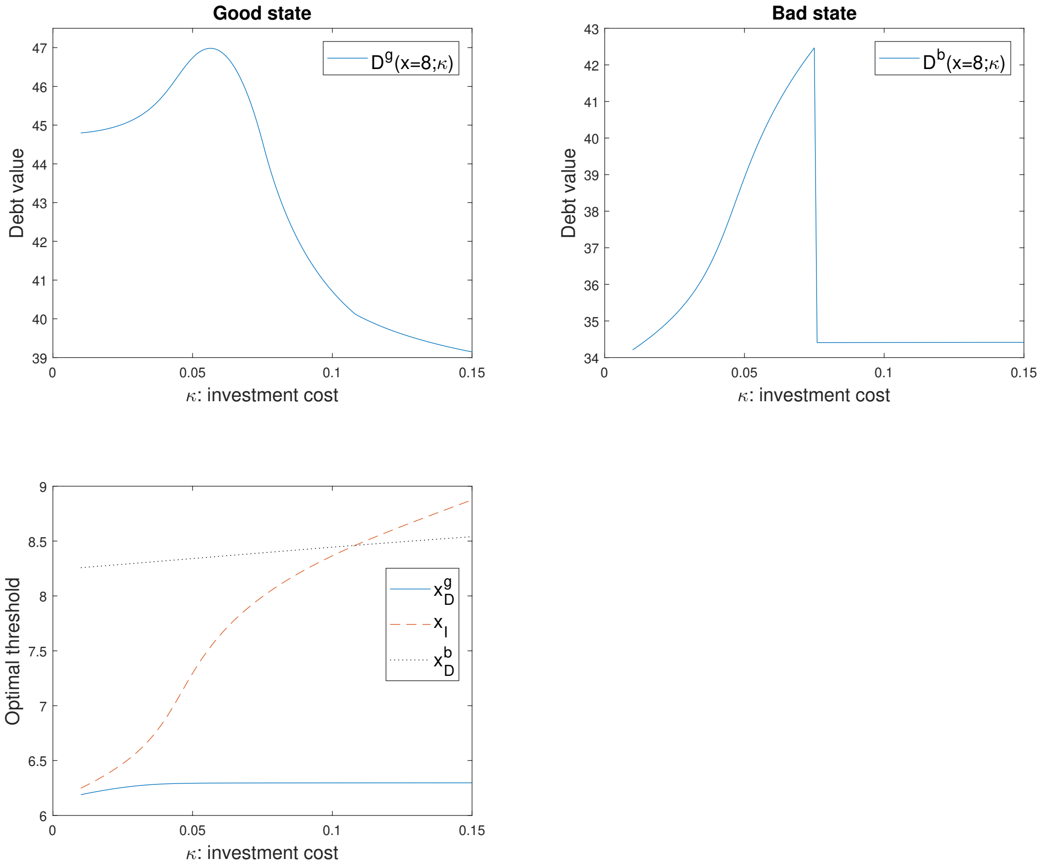

Figure 5: The two panels on the top plot $D^g(x = 8; \kappa)$ and $D^b(x = 8; \kappa)$, respectively. The bottom panel plots the optimal thresholds for different values of κ.

firms' investment decisions do not carry any useful information and therefore, the recovery value for failed firms will be lowest possible. Thus, if κ is reduced to 0.06 so that x_I falls below x^b_D, the information channel increases the debt value unambiguously. Then, the firm value must increase as well.

Figure 5 illustrates the results in the above two cases in an alternative way. We first pick a particular point $x = 8$. We then plot both $D^g(x; \kappa)$ and $D^b(x; \kappa)$ for $\kappa \in [0.01, 0.15]$ in the top two panels. In the bottom panel, we also plot the optimal thresholds x^g_D, x^b_D, and x_I for $\kappa \in [0.01, 0.15]$. Specifically, the bottom panel shows that the investment threshold x_I coincides with $x = 8$ when $\kappa = 0.076$. Also, the point $x = 8$ always lies between x^g_D and x^b_D for any different κ. Thus, due to the information channel, the debt value $D^b(x; \kappa)$, which is the same as the recovery value, increases in κ until κ reaches 0.076. But then, $D^b(\underline{x};\kappa)$ jumps down as soon as κ exceeds 0.076.

If κ increases further beyond 0.076, $D^b(\underline{x};\kappa)$ is almost flat. Here, if the default threshold x_D^g is hypothetically fixed, $D^b(\underline{x};\kappa)$ must be completely flat as discussed in the previous section. But now, as κ increases, x_D^g increases as well. Thus, due to the minor informational role of the default channel, $D^b(\underline{x};\kappa)$ must increase to some extent. But the graph for $D^b(\underline{x};\kappa)$ suggests that such an effect is almost negligible.

The debt value in the good state $D^g(x;\kappa)$ exhibits a clear non-monotone pattern. That is, $D^g(x;\kappa)$ increases in κ until κ reaches 0.056 but then, decreases as κ increases further. This result implies that when κ is low so that the investment threshold lies below the point $x = 8$, the information channel tends to dominate the default channel. But when κ is high so that the investment threshold lies above the point $x = 8$, the default channel tends to dominate the information channel.

4 Empirical Analysis

We have thus far seen that some changes in government policies or economic conditions inducing more investments do not necessarily improve a debt value or a firm value. This result can thus support our motivating empirical fact depicted in Figure 1. However, as mentioned in introduction, this empirical pattern can be also explained by another channel, namely, inefficient overinvestments. Thus, to figure out which channel between the two mechanisms better explains the patterns in Figure 1, we analyze the relationship between the investment rate and the recovery rate in a more refined way. We then present some suggestive empirical evidence that supports our model rather than the overinvestment channel. Before doing so, we briefly discuss the overinvestment mechanism.

4.1 Overinvestment Channel

In general, firms may invest in negative-NPV projects for two reasons. First, the well-known risk-shifting problem can cause investments in risky inefficient projects, as first addressed by Jensen and Meckling (1976). Second, an overconfident manager can invest in a negative-NPV project if she overestimates the project's future profits.

Regarding the risk-shifting problem, imagine an oil drilling company that has a loan of \$10 due next year. But the firm's existing oil reserves will produce only \$8 next year. Accordingly, the firm will go bankrupt with certainty and the bond recovery rate will be 80%. Now suppose the firm has an investment opportunity to explore new oil reserves. If this project succeeds, the firm's total free cash flows will be \$12. If the project fails, the total free cash flows will be \$2, thereby reducing the recovery rate to 20%. Assuming the upside and downside events will occur equally likely, the NPV of the project is negative. Nonetheless, the equityholders will undertake this project because the downside risk will be borne only by the creditors. Hence, the risk-shifting problem can also generate a negative relationship between the ex-ante investments and the ex-post recovery rate of defaulted bonds. We can understand the overconfidence channel similarly.

However, notice that our model and the overinvestment channel actually predict different outcomes that can be tested by data. First, the above argument says that the overinvestment channel causes the negative relationship between the investment rate and the recovery rate within each individual firm. In our model, however, a low recovery rate of good firms is driven by a high investment rate of bad firms. As such, in the next section, we use this key distinctive feature to examine which channel is more consistent with data.

4.2 Suggestive Empirical Evidence

To test the above-mentioned different predictions, we run the following regression:

$$\text{Recovery rate}_{c,t} = \alpha + \beta_1 \times \text{Investment rate}_{c,t-2} + \beta_2 \times \text{Investment rate}_{c-1,t-2}. \tag{18}$$

The explained variable is the issuer-weighted average recovery rate of senior unsecured bonds that defaulted at year t and had a credit rating of c at year $t-1$. We use Moody's credit rating scales for long-term bonds, that is, $\{\text{Aaa}, \text{Aa}, \cdots, \text{B}, \text{Caa-C}\}$, to denote the credit rating c, where we group the ratings from Caa to C together, following the convention. Also, note that we use the credit ratings one year prior to default as the most recent credit ratings for defaulted firms, because firms at default are typically rated C or Ca, which may not be that informative to outside investors.

Moreover, in this regression, we focus on the firms rated Baa or below, because firms rated above Baa rarely default and thus, are irrelevant to our model predictions. Note that the border line between Baa and Ba separates non-investment grade firms from investment grade firms. Thus, the default rate of Baa firms is still low, although not negligible. To deal with this a bit limited data availability, we take the average of the recovery rates over the period from $t-1$ to $t+1$ instead of looking at only one-year period, when calculating the explained variable. Then, to account for these overlapping observations, we use the Newey-West correction with 3-year lags to estimate the standard errors for the regression coefficients.

In addition, our regression uses two explanatory variables. The first one is the issuer-weighted average investment rate of firms at year $t-2$, whose credit rates were c at that year. Here, the investment rate is defined as the total net investments in tangible capital such as plants, property, and equipment divided by the total asset value. Moreover, as in Figure 1, we again use a two-year gap between the investment rate and the recovery rate to account for the fact that investment in tangible capital usually takes more than one year to be complete. The second explanatory variable is the issuer-weighted average investment rate of firms at year $t-2$, whose credit ratings were $c-1$ at that year. Here, $c-1$ is the credit rating immediately below the credit rating c. That is, Ba, B, Caa-C are the credit ratings immediately below Baa, Ba, and B, respectively.

Before proceeding further, we discuss the following issue: One may argue that the credit ratings themselves can simply eliminate information asymmetry in the secondary market. But credit-rating agencies are not always able to estimate firms' financial conditions accurately and thus, cannot remove the imperfect information problem completely. In this regard, in the above regression, we implicitly assume that outside investors still face difficulty differentiating between good firms and bad firms, especially when their credit ratings are adjacent to each other. Hence, the past investment choices of those firms can still help investors identify which firms are in a better condition.

Table 2 summarizes the regression results. The second line in column (1) shows a statistically significant negative relationship between the investment rate of Ba firms and the

Explained Variable =	Baa Recovery Rate		Ba Recovery Rate		B Recovery Rate	
	(1)	(2)	(3)	(4)	(5)	(6)
Baa Investment Rate	−0.570					
	(0.414)					
Ba Investment Rate	−0.507***	−0.753***	−0.112			
	(0.172)	(0.086)	(0.177)			
B Investment Rate			−0.391**	−0.445**	−0.465	
			(0.152)	(0.206)	(0.378)	
Caa-C Investment Rate					−0.278	−0.369
					(0.228)	(0.359)
Constant	63.198***	60.777***	52.421***	51.244***	56.249***	48.301***
	(8.879)	(5.475)	(7.576)	(5.514)	(5.957)	(8.787)
Observations	18	21	23	23	21	21
R^2	0.225	0.219	0.210	0.202	0.225	0.046
Note:				*$p < 0.1$; **$p < 0.05$; ***$p < 0.01$		

Table 2: This table presents the coefficients and t-values of the regression described in (18). The data on the investment rates, credit ratings, and recovery rates come from Compustat and Moody's Default and Recovery Database. As Compustat uses S&P's credit rating scales, we convert them to the equivalent credit ratings of Moody's. The dataset covers the period from 1985 to 2014.

recovery rate of Baa firms. Specifically, fixing the investment rate of Baa firms, a 1% higher investment rate of Ba firms predicts a 0.51% lower recovery rate of Baa firms two years later. The left panel in Figure 6 visualizes the relationship between these two variables. Of course, in this graph, the slope of the best-fitting line corresponds to the coefficient in a regression that uses only the investment rate of Ba firms as an explanatory, as shown in column (2). Meanwhile, the first line in column (1) shows that the investment rate of Baa firms and the recovery rate of the same group of firms have no significant relationship with each other. That is, although the regression coefficient is still negative, it is statistically insignificant.

Column (3) exhibits the same patterns between Ba firms and B firms. Specifically, the investment rate of B firms is negatively related with the recovery rate of Ba firms, which is again statistically significant. The right panel in Figure 6 depicts this relationship. However, within Ba firms, the investment rate and the recovery rate do not show any significant relationship with each other.

All these results are more consistent with our model rather than the overinvestment mechanism. First, the negative relationship between the investment rate and the recovery rate across adjacently-rated firms supports our model predictions pretty well. Yet the statistically

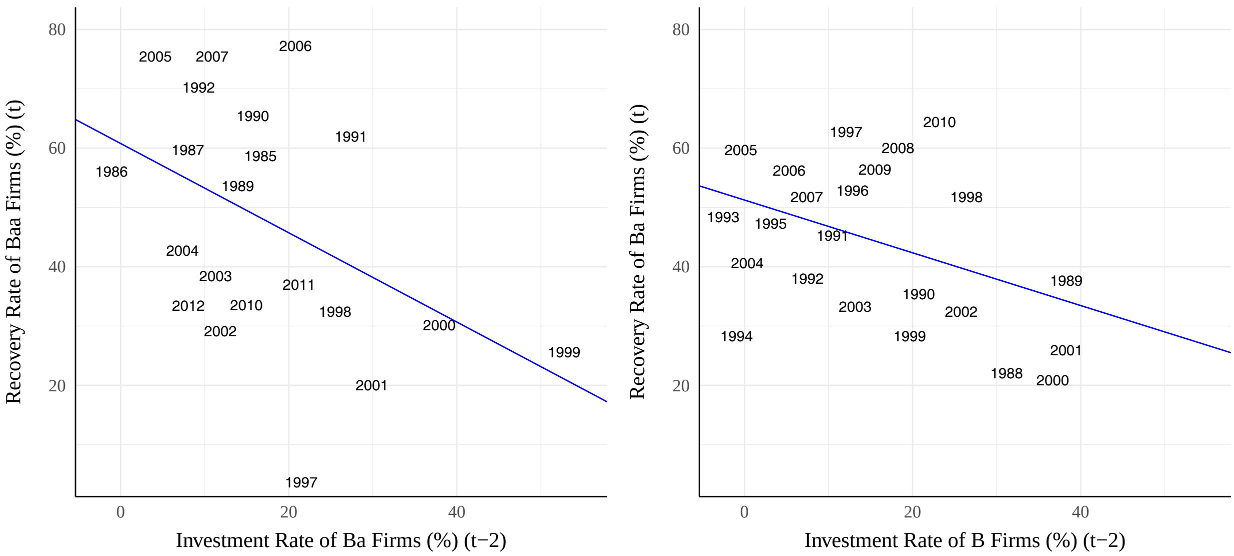

Figure 6: The left panel plots the relationship between the investment rate of Ba firms at time $t-2$ and the recovery rate of Baa firm at time t over the period from 1985 to 2014. The right panel plots the relationship between the investment rate of B firms at time $t-2$ and the recovery rate of Ba firm at time t over the same period.

insignificant relationship between those two variables within equally-rated firms does not match well with the overinvestment channel.

However, column (5) does not show any significant relationship between the recovery rate of B firms and the investment rate of Caa-C firms. This result, which looks inconsistent with our model predictions, may have obtained for several reasons. First, firms are rarely rated Caa or below two years prior to default. Thus, the data on the investment rates of C-Caa firms may not be that reliable. Second, we can actually explain the insignificant relationship between the two variables using our model to some extent. Note that in our model, firms whose asset qualities are severely low are not sensitive to the changes in government policies or other economic conditions that affect the incentives for new investments. If firms rated B or below belong to this type of firms, our model does not predict any meaningful relationship between Caa-C firms and B firms. In this regard, we can take the results in columns (1) and (3) more seriously than the result in column (5), because Baa firms or Ba firms seem to fit best those relatively good firms in our model.

Lastly, one may argue that the overinvestment channel can also explain the negative relationship between the investment rate of low-rated firms and the recovery rate of high-rated firms. Specifically, when the demand for failed assets is not perfectly elastic, asset liquidation by any failed firms causes a price impact on the assets of other firms. This spillover effect can thus explain the above negative relationship between adjacently-rated firms. Nonetheless, as mentioned above, the insignificant relationship between the investment rate and the recovery rate within equally-rated firms is still questionable to adopt the overinvestment channel. In this regard, we can keep our viewpoint that the empirical results in Table 2 better support our model.

5 Conclusion

We introduce a new information channel of debt overhang. When considering only debt overhang, policies that target an increase in investment is unambiguously beneficial to all agents and increase firm value. However, if the secondary asset market suffers from information asymmetry, increasing investment may decrease the informational value of debt overhang.

Because firms with higher asset quality in bankruptcy retain a larger fraction of assets to signal their quality to secondary asset buyers who are better at managing those firm assets, higher quality assets actually have less liquidity and do not realize the full gains from trade. However, prior to bankruptcy, equity decides whether a firm invests in a new project. Thus, for firms to invest in new projects, equity must expect a positive return. Therefore, in bankruptcy, secondary asset market buyers can indirectly infer a range of firm quality based on their investment decisions prior to default, giving rise to the information channel of debt overhang.

6 Appendix

6.1 Omitted Proof in Section 3.2.1

The retention strategy $f(x)$ given by (15) satisfies that, for any $y \in [x_D^g, x_I)$,

$$
\begin{aligned}
& f(x)\alpha x + (1 - f(x))\beta x - [f(y)\alpha x + (1 - f(y))\beta y] \\
& = \int_y^x \frac{\partial}{\partial z}[f(z)\alpha x + (1 - f(z))\beta z]dz \\
& = \int_y^x [(\alpha - \beta)zf'(z) + \beta(1 - f(z)) + f'(z)\alpha(x - z)]dz \\
& = \int_y^x f'(z)\alpha(x - z)dz, \quad \text{by the FOC (14)}, \\
& \geq 0, \quad \text{because } f'(z) \geq 0.
\end{aligned}
$$

Hence, the above retention strategy indeed solves the maximization problem in (13).

Now, suppose that the creditor of type $x \in [x_D^g, x_I)$ retains a fraction ξ of her asset, where there is no $y \in [x_D^g, x_I)$ such that $\xi = f(y)$. Then, because $f(x_D^g) = 0$ and $f(\cdot)$ increases over $[x_D^g, x_I)$, ξ must be larger than $f(x)$. Thus, we have

$$
\begin{aligned}
\xi\alpha x + (1 - \xi)\beta x_D^g &\leq \xi\alpha x + (1 - \xi)\beta x \\
&< f(x)\alpha x + (1 - f(x))\beta x,
\end{aligned}
$$

implying the above creditor does not have any incentives to choose ξ as the retention ratio. We have therefore completed the proof.

6.2 Almost Closed-Form Solutions

(This section will be updated soon.) In this section, for any given $\{x_D^g, x_I, x_D^b\}$, we first solve for the equity and debt values in closed form. We then compute equilibrium thresholds $\{x_D^g, x_I, x_D^b\}$ numerically by solving the conditions satisfied by these thresholds in equilibrium. In what follows, replace ϕ_g by ϕ and set $\phi_b = 0$.

First, consider the case of $x_D^g < x_I < x_D^b$. In this case, we can solve for $E^g(x)$ and $E^b(x)$

using a standard technique for the system of ordinary differential equations. Specifically, consider the following matrix:

$$M = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2\sigma^{-2}(r+\phi_g) & -2\sigma^{-2}\phi_g & -2\sigma^{-2}\mu_H + 1 & 0 \\ -2\sigma^{-2}\phi_b & 2\sigma^{-2}(r+\phi_b) & 0 & -2\sigma^{-2}\mu_L + 1 \end{pmatrix}.$$

Then, let λ_1 and λ_2 be the two negative eigenvalues for M. We postpone to show that M has indeed two negative eigenvalues and two positive eigenvalues. Also, let v_{1g} and v_{1g} be the first two components of the eigenvector for λ_1. Similarly, let v_{2g} and v_{2g} be the first two components of the eigenvector for λ_2. Then, the equity value is given by

$$E^g(x) = \begin{cases} E^{g,1}(x) = -\frac{(1-\pi)c}{r+\phi_g} + \frac{x}{r+\phi_g-\mu_L} + A_1 x^{\eta_1} + A_2 x^{\eta_2}, & \text{if } x_D^g \le x \le x_I \\ E^{g,2}(x) = -\frac{(1-\pi)c}{r+\phi_g} + \frac{(1-\kappa)x}{r+\phi_g-\mu_H} + A_3 x^{\eta_3} + A_4 x^{\eta_4}, & \text{if } x_I \le x \le x_D^b \\ E^{g,3}(x) = -\frac{(1-\pi)c}{r} + F^g(x) + A_5 v_{1g} x^{\lambda_1} + A_6 v_{2g} x^{\lambda_2}, & \text{if } x_D^b \le x \end{cases}$$

and

$$E^b(x) = \begin{cases} E^{b,1} = 0, & \text{if } x \le x_D^b \\ E^{b,2}(x) = -\frac{(1-\pi)c}{r} + F^b(x) + A_5 v_{1b} x^{\lambda_1} + A_6 v_{2b} x^{\lambda_2}, & \text{if } x_D^b \le x, \end{cases}$$

where

$$\begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \sigma^{-2}\left(-\mu_L + \frac{\sigma^2}{2} \pm \sqrt{\left(\mu_L - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(r+\phi_g)}\right),$$

$$\begin{bmatrix} \eta_3 \\ \eta_4 \end{bmatrix} = \sigma^{-2}\left(-\mu_H + \frac{\sigma^2}{2} \pm \sqrt{\left(\mu_H - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(r+\phi_g)}\right).$$

Now, note that $\{x_D^g, x_I, x_D^b, A_1, A_2, \cdots, A_6\}$ must satisfy the following nine conditions:

$$E^{g,1}(x_D^g) = 0, \quad E_x^{g,1}(x_D^g) = 0, \quad E^{g,1}(x_I) = E^{g,2}(x_I), \quad E_x^{g,1}(x_I) = E_x^{g,2}(x_I) = \frac{\kappa}{\delta},$$
$$E^{g,2}(x_D^b) = E^{g,3}(x_D^b), \quad E_x^{g,2}(x_D^b) = E_x^{g,3}(x_D^b), \quad E^{b,2}(x_D^b) = 0, \quad E_x^{b,2}(x_D^b) = 0.$$

We solve this system of equations numerically.

Let us now show that M has two negative eigenvalues and two positive eigenvalues. Specifically, $f(\lambda) := \det(M - \lambda I)$ is given by

$$f(\lambda) = \lambda^2(-2\sigma^{-2}\mu_H + 1 - \lambda)(-2\sigma^{-2}\mu_L + 1 - \lambda) + \lambda 2\sigma^{-2}(r + \phi_b)(-2\sigma^2\mu_H + 1 - \lambda)+$$
$$\lambda 2\sigma^{-2}(r + \phi_g)(-2\sigma^{-2}\mu_L + 1 - \lambda) + 4\sigma^{-4}(r + \phi_g)(r + \phi_b) - 4\sigma^{-4}\phi_g\phi_b.$$

Then, from some algebra, we have

$$f(\lambda) = [\lambda(-2\sigma^{-2}\mu_L + 1 - \lambda) + 2\sigma^{-2}r][\lambda(-2\sigma^{-2}\mu_H + 1 - \lambda) + 2\sigma^{-2}r]+$$
$$2\sigma^{-2}\phi_b[\lambda(-2\sigma^{-2}\mu_H + 1 - \lambda) + 2\sigma^{-2}r] + 2\sigma^{-2}\phi_g[\lambda(-2\sigma^{-2}\mu_L + 1 - \lambda) + 2\sigma^{-2}r].$$

Now, let λ_H be the negative solution to

$$\lambda(-2\sigma^{-2}\mu_H + 1 - \lambda) + 2\sigma^{-2}r = 0.$$

Also, let λ_L be the positive solution to

$$\lambda(-2\sigma^{-2}\mu_L + 1 - \lambda) + 2\sigma^{-2}r = 0.$$

Then, using the property that $\mu_L < \mu_H$, we know that

$$\lambda_H(-2\sigma^{-2}\mu_L + 1 - \lambda_H) + 2\sigma^{-2}r = 2\lambda_H\sigma^{-2}(\mu_H - \mu_L) < 0,$$

$$\lambda_L(-2\sigma^{-2}\mu_H + 1 - \lambda_L) + 2\sigma^{-2}r = -2\lambda_L\sigma^{-2}(\mu_H - \mu_L) < 0.$$

This result implies

$$f(\lambda_H) < 0 \quad \text{and} \quad f(\lambda_L) < 0.$$

Also, note that $f(0) > 0$. Therefore, $f(\lambda)$ must have two negative roots and two positive roots.

On the other hand, the debt value is given by

$$D^g(x) = \begin{cases} D^{g,1}(x) = \frac{c}{r+\phi_g} + \frac{\phi_g \alpha H^b x}{r+\phi_g-\mu_L} + \frac{\phi_g(\beta-\alpha)H^b(x_D^g)^\xi x^{1-\xi}}{r+\phi_g-\mu_L(1-\xi)+\frac{\sigma^2}{2}(1-\xi)\xi} + B_1 x^{\eta_1} + B_2 x^{\eta_2}, & \text{if } x_D^g \le x \le x_I \\ D^{g,2}(x) = \frac{c}{r+\phi_g} + \frac{\phi_g \alpha H^b x}{r+\phi_g-\mu_H} + \frac{\phi_g(\beta-\alpha)H^b(x_I)^\xi x^{1-\xi}}{r+\phi_g-\mu_H(1-\xi)+\frac{\sigma^2}{2}(1-\xi)\xi} + B_3 x^{\eta_3} + B_4 x^{\eta_4}, & \text{if } x_I \le x \le x_D^b \\ D^{g,3}(x) = \frac{c}{r} + B_5 v_{1g} x^{\lambda_1} + B_6 v_{2g} x^{\lambda_2}, & \text{if } x_D^b \le x, \end{cases}$$

where $\xi = \frac{\beta}{\beta-\alpha}$ and

$$D^b(x) = \begin{cases} D^{b,1}(x) = R(x), & \text{if } x_D^g \le x < x_D^b \\ D^{b,2}(x) = \frac{c}{r} + B_5 v_{1b} x^{\lambda_1} + B_6 v_{2b} x^{\lambda_2}, & \text{if } x_D^b \le x. \end{cases}$$

For clarification, the recovery value $R(x)$ is given by

$$R(x) = \begin{cases} \left[1 - \left(\frac{x_D^g}{x}\right)^{\frac{\beta}{\beta-\alpha}}\right] \alpha H^b x + \left(\frac{x_D^g}{x}\right)^{\frac{\beta}{\beta-\alpha}} \beta H^b x, & \text{if } x_D^g \le x < x_I \\ \left[1 - \left(\frac{x_I}{x}\right)^{\frac{\beta}{\beta-\alpha}}\right] \alpha H^b x + \left(\frac{x_I}{x}\right)^{\frac{\beta}{\beta-\alpha}} \beta H^b x, & \text{if } x_I \le x < x_D^b. \end{cases}$$

Now, note that $\{B_1, B_2, \cdots, B_6\}$ must satisfy the following six conditions:

$$D^{g,1}(x_D^g) = \beta H^g x_D^g, \quad D^{g,1}(x_I) = D^{g,2}(x_I), \quad D_x^{g,1}(x_I) = D_x^{g,2}(x_I),$$
$$D^{g,2}(x_D^b) = D^{g,3}(x_D^b), \quad D_x^{g,2}(x_D^b) = D_x^{g,3}(x_D^b), \quad D^{b,2}(x_D^b) = \beta H^b x_D^b.$$

We can solve this system of linear equations in closed form.

Now, let us consider the case of $x_D^g < x_D^b < x_I$. In this case, we first consider the following matrix:

$$N = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2\sigma^{-2}(r+\phi_g) & -2\sigma^{-2}\phi_g & -2\sigma^{-2}\mu_L + 1 & 0 \\ -2\sigma^{-2}\phi_b & 2\sigma^{-2}(r+\phi_b) & 0 & -2\sigma^{-2}\mu_L + 1 \end{pmatrix}.$$

We can show that N has two negative real eigenvalues and two real positive eigenvalues similarly as above. Thus, let $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ be those four eigenvalues for N. Also, for each

$i = \{1, 2, 3, 4\}$, let v_{ig} and v_{ib} be the first two components of the eigenvector for λ_i.

Now, consider another matrix:

$$H = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2\sigma^{-2}(r+\phi_g) & -2\sigma^{-2}\phi_g & -2\sigma^{-2}\mu_H + 1 & 0 \\ -2\sigma^{-2}\phi_b & 2\sigma^{-2}(r+\phi_b) & 0 & -2\sigma^{-2}\mu_L + 1 \end{pmatrix}.$$

Again, we can similarly show that H has two negative eigenvalues and two positive eigenvalues. Thus, let $\{\lambda_5, \lambda_6\}$ be the two negative eigenvalues for H. Also, for each $i \in \{5, 6\}$, let v_{ig} and v_{ib} be the first two components of the eigenvector for λ_i.

Then, $E^g(x)$ is given by

$$\begin{cases} E^{g,1}(x) = -\frac{(1-\pi)c}{r+\phi_g} + \frac{x}{r+\phi_g-\mu_L} + A_1 x^{\eta_1} + A_2 x^{\eta_2}, & \text{if } x_D^g \leq x \leq x_D^b \\ E^{g,2}(x) = -\frac{(1-\pi)c}{r} + C_{1g}x + A_3 v_{1g} x^{\lambda_1} + A_4 v_{2g} x^{\lambda_2} + A_5 v_{3g} x^{\lambda_3} + A_6 v_{4g} x^{\lambda_4}, & \text{if } x_D^b \leq x \leq x_I \\ E^{g,3}(x) = -\frac{(1-\pi)c}{r} + C_{2g}x + A_7 v_{5g} x^{\lambda_5} + A_8 v_{6g} x^{\lambda_6}, & \text{if } x_I \leq x \end{cases}$$

and $E^b(x)$ is given by

$$\begin{cases} E^{b,1}(x) = 0, & \text{if } x \leq x_D^b \\ E^{b,2}(x) = -\frac{(1-\pi)c}{r} + C_{1b}x + A_3 v_{1b} x^{\lambda_1} + A_4 v_{2b} x^{\lambda_2} + A_5 v_{3b} x^{\lambda_3} + A_6 v_{4b} x^{\lambda_4}, & \text{if } x_D^b \leq x \leq x_I, \\ E^{b,3}(x) = -\frac{(1-\pi)c}{r} + C_{2b}x + A_7 v_{5b} x^{\lambda_5} + A_8 v_{6b} x^{\lambda_6}, & \text{if } x_I \leq x \end{cases}$$

where

$$C_{1g} = \frac{r + \phi_b - \mu_L + \phi_g\gamma}{(r-\mu_L)(r+\phi_g+\phi_b-\mu_L)}, \; C_{1b} = \frac{\gamma(r+\phi_g-\mu_L)+\phi_b}{(r-\mu_L)(r+\phi_g+\phi_b-\mu_L)},$$

$$C_{2g} = \frac{(1-\kappa)(r+\phi_b-\mu_L)+\phi_g\gamma}{(r-\mu_H)(r+\phi_b-\mu_L)+\phi_g(r-\mu_L)}, \; C_{2b} = \frac{\gamma(r+\phi_g-\mu_H)+\phi_b(1-\kappa)}{(r-\mu_L)(r+\phi_g-\mu_H)+\phi_b(r+\phi_g-\mu_H)},$$

$$\begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \sigma^{-2}\left(-\mu_L + \frac{\sigma^2}{2} \pm \sqrt{\left(\mu_L - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(r+\phi_g)}\right).$$

Again, $\{x_D^g, x_I, x_D^b, A_1, \cdots, A_8\}$ satisfy the following eleven conditions:

$$E^{g,1}(x_D^g) = 0, \quad E_x^{g,1}(x_D^g) = 0, \quad E^{g,1}(x_D^b) = E^{g,2}(x_D^b), \quad E_x^{g,1}(x_D^b) = E_x^{g,2}(x_D^b),$$
$$E^{g,2}(x_I) = E^{g,3}(x_I), \quad E_x^{g,2}(x_I) = E_x^{g,3}(x_I) = \frac{\kappa}{\delta}, \quad E^{b,2}(x_D^b) = 0, \quad E_x^{b,2}(x_D^b) = 0,$$
$$E^{b,2}(x_I) = E^{b,3}(x_I), \quad E_x^{b2}(x_I) = E_x^{b,3}(x_I).$$

We solve this system of equations numerically.

Now, let us compute the debt value. Note that $D^g(x)$ is given by

$$\begin{cases} D^{g,1}(x) = \frac{c}{r+\phi_g} + \frac{\phi_g \alpha H^b x}{r+\phi_g-\mu_L} + \frac{\phi_g(\beta-\alpha)H^b (x_D^g)^{\xi} x^{1-\xi}}{r+\phi_g-\mu_L(1-\xi)+\frac{\sigma^2}{2}(1-\xi)\xi} + B_1 x^{\eta_1} + B_2 x^{\eta_2}, & \text{if } x_D^g \le x \le x_D^b \\ D^{g,2}(x) = \frac{c}{r} + B_3 v_{1g} x^{\lambda_1} + B_4 v_{2g} x^{\lambda_2} + B_5 v_{3g} x^{\lambda_3} + B_6 v_{4g} x^{\lambda_4}, & \text{if } x_D^b \le x \le x_I \\ D^{g,3}(x) = \frac{c}{r} + B_7 v_{5g} x^{\lambda_5} + B_8 v_{6g} x^{\lambda_6}, & \text{if } x_I \le x, \end{cases}$$

where $\xi = \frac{\beta}{\beta-\alpha}$. Also, $D^b(x)$ is given by

$$\begin{cases} D^{b,1}(x) = R(x), & \text{if } x_D^g \le x < x_D^b \\ D^{b,2}(x) = \frac{c}{r} + B_3 v_{1b} x^{\lambda_1} + B_4 v_{2b} x^{\lambda_2} + B_5 v_{3b} x^{\lambda_3} + B_6 v_{4b} x^{\lambda_4}, & \text{if } x_D^b \le x \le x_I \\ D^{b,3}(x) = \frac{c}{r} + B_7 v_{5b} x^{\lambda_5} + B_8 v_{6b} x^{\lambda_6}, & \text{if } x_I \le x, \end{cases}$$

Here, $R(x)$ is given by

$$R(x) = \left[1 - \left(\frac{x_D^g}{x}\right)^{\frac{\beta}{\beta-\alpha}}\right] \alpha H^b x + \left(\frac{x_D^g}{x}\right)^{\frac{\beta}{\beta-\alpha}} \beta H^b x.$$

Lastly, note that $\{B_1, \cdots, B_8\}$ satisfy the following eight conditions:

$$D^{g,1}(x_D^g) = \beta H^g x_D^g, \quad D^{g,1}(x_D^b) = D^{g,2}(x_D^b), \quad D_x^{g,1}(x_D^b) = D_x^{g,2}(x_D^b), \quad D^{g,2}(x_I) = D^{g,3}(x_I),$$
$$D_x^{g,2}(x_I) = D_x^{g,3}(x_I), \quad D^{b,2}(x_D^b) = \beta H^b x_D^b, \quad D^{b,2}(x_I) = D^{b,3}(x_I), \quad D_x^{b2}(x_I) = D_x^{b,3}(x_I).$$

We can solve this system of linear equations in closed form.

References

Chen, H. (2010). Macroeconomic conditions and the puzzles of credit spreads and capital structure. *The Journal of Finance*, 65(6):2171–2212.

Chen, H. and Manso, G. (2017). Macroeconomic risk and debt overhang. *Review of Corporate Finance Studies*, 6(1):1–38.

Diamond, D. W. and He, Z. (2014). A theory of debt maturity: the long and short of debt overhang. *The Journal of Finance*, 69(2):719–762.

Gertner, R. and Scharfstein, D. (1991). A theory of workouts and the effects of reorganization law. *The Journal of Finance*, 46(4):1189–1222.

Hackbarth, D. and Mauer, D. C. (2012). Optimal priority structure, capital structure, and investment. *The Review of Financial Studies*, 25(3):747–796.

Hackbarth, D., Miao, J., and Morellec, E. (2006). Capital structure, credit risk, and macroeconomic conditions. *Journal of Financial Economics*, 82(3):519–550.

Han, S. and Zhou, X. (2014). Informed bond trading, corporate yield spreads, and corporate default prediction. *Management Science*, 60(3):675–694.

He, Z. (2011). A model of dynamic compensation and capital structure. *Journal of Financial Economics*, 100(2):351–366.

He, Z. and Xiong, W. (2012). Rollover risk and credit risk. *The Journal of Finance*, 67(2):391–430.

Hennessy, C. A. (2004). Tobin's q, debt overhang, and investment. *The Journal of Finance*, 59(4):1717–1742.

Hennessy, C. A., Levy, A., and Whited, T. M. (2007). Testing q theory with financing frictions. *Journal of Financial Economics*, 83(3):691–717.

Huang, J.-Z. and Huang, M. (2012). How much of the corporate-treasury yield spread is due to credit risk? *The Review of Asset Pricing Studies*, 2(2):153–202.

Jensen, M. C. and Meckling, W. H. (1976). Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of financial economics*, 3(4):305–360.

Kedia, S. and Zhou, X. (2014). Informed trading around acquisitions: Evidence from corporate bonds. *Journal of Financial Markets*, 18:182–205.

Leland, H. E. (1994). Corporate debt value, bond covenants, and optimal capital structure. *The journal of finance*, 49(4):1213–1252.

Leland, H. E. and Pyle, D. H. (1977). Informational asymmetries, financial structure, and financial intermediation. *The journal of Finance*, 32(2):371–387.

Mello, A. S. and Parsons, J. E. (1992). Measuring the agency cost of debt. *The Journal of Finance*, 47(5):1887–1904.

Miao, J. (2005). Optimal capital structure and industry dynamics. *The Journal of finance*, 60(6):2621–2659.

Miller, M. H. (1977). Debt and taxes. *the Journal of Finance*, 32(2):261–275.

Moyen, N. (2007). How big is the debt overhang problem? *Journal of Economic Dynamics and Control*, 31(2):433–472.

Myers, S. C. (1977). Determinants of corporate borrowing. *Journal of Financial Economics*, 5(2):147–175.

Stulz, R. and Johnson, H. (1985). An analysis of secured debt. *Journal of financial Economics*, 14(4):501–521.

Sundaresan, S., Wang, N., and Yang, J. (2015). Dynamic investment, capital structure, and debt overhang. *The Review of Corporate Finance Studies*, 4(1):1–42.

Titman, S. and Tsyplakov, S. (2007). A dynamic model of optimal capital structure. *Review of Finance*, 11(3):401–451.

Wittenberg-Moerman, R. (2008). The role of information asymmetry and financial reporting quality in debt trading: Evidence from the secondary loan market. *Journal of Accounting and Economics*, 46(2-3):240–260.